



MODULE 3

MATHEMATICS FOR IPMAT INTERVIEW

Calculus · Graphs · Probability · Algebra · Statistics

Mathematics questions are asked in almost every IPMAT interview panel. Panelists test conceptual clarity, not just formulae. This module covers every mathematical topic asked across IIM Indore, Rohtak, and Ranchi — with model answers, key theorems, graph descriptions, and real-life application questions.

WHAT'S INSIDE THIS MODULE

■ Graphs & Functions

30+ graph questions with key features described

} Calculus

Differentiation, integration, applications

■ Probability

Concepts, Bayes theorem, actual Q&A;

■ Statistics

Mean / Median / Mode, standard deviation

■ Algebra

Matrices, determinants, sequences

■ Application Q&A;

Real-life application questions answered

1. ESSENTIAL FORMULA SHEET

You must be able to state **and** derive these on demand. Panelists often ask: 'What is the formula? Can you prove it?'

Differentiation — Key Derivatives

FUNCTION $f(x)$	DERIVATIVE $f'(x)$	NOTES
x^n	$n \cdot x^{n-1}$	Power rule — most fundamental
e^x	e^x	Unique: derivative equals the function itself
$\ln x$	$1/x$	Important for log-graph questions
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$\sin^{-1} x$	$1/\sqrt{1-x^2}$	Inverse trig
$\tan^{-1} x$	$1/(1+x^2)$	Inverse trig
$u \cdot v$	$u'v + uv'$	Product rule
u/v	$(u'v - uv')/v^2$	Quotient rule

Integration — Key Integrals

FUNCTION	INTEGRAL $\int f(x) dx$	NOTES
x^n	$x^{n+1}/(n+1) + C$	$n \neq -1$
$1/x$	$\ln x + C$	Asked very frequently
e^x	$e^x + C$	
$\sin x$	$-\cos x + C$	
$\cos x$	$\sin x + C$	
$\ln x$	$x \ln x - x + C$	Integration by parts

Probability Formulas

CONCEPT	FORMULA
$P(A \cup B)$	$P(A) + P(B) - P(A \cap B)$
$P(A B)$ Conditional	$P(A \cap B) / P(B)$
Bayes' Theorem	$P(A B) = P(B A) \cdot P(A) / P(B)$
Combination $C(n,r)$	$n! / [r! \cdot (n-r)!]$
Permutation $P(n,r)$	$n! / (n-r)!$
Independent events	$P(A \cap B) = P(A) \cdot P(B)$
Mutually exclusive	$P(A \cap B) = 0$

Statistics

MEASURE	FORMULA / DEFINITION
Arithmetic Mean	Sum of all values \div number of values
Median	Middle value when data is ordered (average of two middle values for even n)
Mode	Most frequently occurring value
Standard Deviation	$\sqrt{[\Sigma(x - \mu)^2 / n]}$ — measures spread around the mean
Variance	σ^2 — square of standard deviation
$AM \geq GM \geq HM$	Fundamental inequality between the three means

2. GRAPH QUESTIONS (Most Commonly Asked)

You will be asked to draw or describe a graph on demand. Practice sketching each one on paper. In online interviews, describe key features verbally: **domain, range, asymptotes, intercepts, shape, behaviour at extremes.**

Graph of $y = x^2$

Upward parabola, vertex at origin, symmetric about the y-axis. $y \geq 0$ always.

Graph of $y = -x^2$

Downward parabola, maximum at origin. Used to illustrate maxima.

Graph of $y = x^3$

S-shaped curve through origin. Inflection point at origin. Odd function.

Graph of $y = e^x$

Always positive. Passes through (0, 1). As $x \rightarrow -\infty$, $y \rightarrow 0$ (asymptote at $y = 0$).

Graph of $y = \ln x$

Defined only for $x > 0$. Passes through (1, 0). As $x \rightarrow 0^+$, $y \rightarrow -\infty$.

Graph of $y = |x|$

V-shape, vertex at origin. Not differentiable at $x = 0$.

Graph of $y = |x - 1|$

V-shape shifted right — vertex at (1, 0).

Graph of $y = |2 - x|$

Vertex at (2, 0). For $x \leq 2$: $y = 2 - x$. For $x > 2$: $y = x - 2$.

Graph of $y = 1/x$

Hyperbola. Two branches in Q1 and Q3. Asymptotes: $x = 0$ and $y = 0$.

Graph of $y = \sin x$

Oscillates between -1 and $+1$. Period = 2π . Passes through (0, 0).

**Graph of $y = \cos x$**

Oscillates between -1 and $+1$. Starts at $(0, 1)$. Period = 2π .

Graph of $y = \tan x$

Period = π . Asymptotes at $x = \pi/2, 3\pi/2, \dots$ Passes through $(0, 0)$.

Graph of $x^2 + y^2 = r^2$

Circle centred at origin, radius r .

Graph of $y^2 = 4ax$

Parabola opening rightward. Vertex at origin. Focus at $(a, 0)$.

Graph of $y = -x^2 + 4$

Downward parabola shifted up. Passes through $(0, 4)$; roots at $x = \pm 2$.

Graph of $y = x^2 - 4$

Upward parabola. Roots at $x = \pm 2$. Vertex at $(0, -4)$.

Graph of $y = \log x$

Same as $y = \ln x$ (natural log) or log base 10. Defined only for $x > 0$.

Graph of $y = x + 1/x$

For $x > 0$: minimum value is 2 at $x = 1$. For $x < 0$: maximum is -2 .

Graph of $y = e^{(x^2)}$

Symmetric about y -axis. Always positive. Minimum at $(0, 1)$.

Graph of Graph of $4x + 3y \leq 12$

Shade the half-plane below the line $4x + 3y = 12$. Intercepts: $(3, 0)$ and $(0, 4)$.

3. MATHEMATICS Q&A; — ACTUAL INTERVIEW QUESTIONS

Q: What is a derivative and what is its geometric meaning?

◆ A derivative is the rate of change of a function with respect to its variable. Geometrically, $f'(x)$ at a point P represents the slope of the tangent line to the curve $y = f(x)$ at P .

Q: What is integration and how does it differ from differentiation?

◆ Differentiation finds the instantaneous rate of change (slope). Integration finds the accumulated area under a curve. They are inverse operations — the Fundamental Theorem of Calculus links them: $\int f'(x) dx = f(x) + C$.

Q: What are the practical applications of differentiation and integration?

◆ Differentiation: finding velocity from displacement, marginal cost in economics, optimisation (maxima / minima), slope of demand curves. Integration: total revenue from marginal revenue, area under curves, volume of solids, computing the path of rivers using calculus.

Q: How would you count the hairs on your head using integration?

◆ Model the scalp as a 2D surface. Use integration to find the total area of the scalp. Multiply by hair density (hairs per cm^2), estimated by counting hairs on a 1 cm^2 patch. ■ density dA over the scalp surface gives the total hair count.

Q: What is the value of $\int |x| dx$ from -1 to 1 ?

◆ Split the integral: $\int_{-1}^1 (-x) dx = [-x^2/2]_{-1}^1 = 1/2$. $\int_{-1}^1 x dx = [x^2/2]_{-1}^1 = 1/2$. Total = $1/2 + 1/2 = 1$.

Q: Can a function be continuous but not differentiable? Give an example.

◆ Yes. $f(x) = |x|$ is continuous everywhere but not differentiable at $x = 0$, because the slope changes abruptly from -1 to $+1$ — left and right derivatives differ.

Q: What is Bayes' Theorem? Give a practical example.

◆ $P(A|B) = P(B|A) \cdot P(A) / P(B)$. Example: a disease affects 1% of the population; the test is 95% accurate. If you test positive, $P(\text{disease}|\text{positive}) \approx 16\%$ — the low base rate dramatically reduces the probability despite high test accuracy.

Q: Probability of getting at most 1 head in 3 coin tosses?

◆ $P(0 \text{ heads}) = (1/2)^3 = 1/8$. $P(1 \text{ head}) = C(3,1) \times (1/2)^3 = 3/8$. $P(\text{at most 1 head}) = 1/8 + 3/8 = 4/8 = 1/2$.

Q: You walk for 15 minutes randomly between 4 pm and 5 pm. Your friend does the same. Probability you meet?

◆ Geometric probability. Sample space = $60 \times 60 = 3600$. Favourable region: $|x - y| \leq 15$. Area = $3600 - 2 \times (45^2/2) = 1575$. $P = 1575 / 3600 = 7/16 \approx 0.4375$.

Q: Maximum and minimum value of $(x + 1/x)$?

◆ For $x > 0$: by AM–GM, $x + 1/x \geq 2$. Minimum = 2 at $x = 1$. No finite maximum. For $x < 0$: $x + 1/x \leq -2$. Maximum = -2 at $x = -1$.

Q: What is a skew-symmetric determinant and what is its value?

◆ A skew-symmetric matrix satisfies $A^T = -A$ (all diagonal entries = 0). For any odd-order skew-symmetric matrix, $\det(A) = 0$.

Q: Sum of the first 100 odd numbers?

◆ Sum of first n odd numbers = n^2 . For $n = 100$: sum = 10,000.

Q: How can you cut a cake into 8 pieces with only 3 cuts?

◆ Cut 1 and Cut 2: two perpendicular cuts through the top \rightarrow 4 pieces. Cut 3: one horizontal cut through the middle \rightarrow doubles each piece \rightarrow 8 pieces.

Q: Why is π equal to 22/7?

◆ π is irrational — 22/7 is a close approximation (≈ 3.14286 vs $\pi \approx 3.14159$), not an equality. Archimedes established bounds $223/71 < \pi < 22/7$ using inscribed and circumscribed polygons.

Q: What does a correlation of 0 mean?

◆ $r = 0$ means there is no **linear** relationship between the variables. A non-linear relationship (e.g., quadratic) may still exist.

Q: Integrate $\log x$.

◆ $\int \ln x dx = x \ln x - x + C$. Use integration by parts: $u = \ln x$, $dv = dx$.

Q: Differentiate $y = |x|$ at $x = 0$.

◆ Not differentiable at $x = 0$. Left-hand derivative = $\lim_{h \rightarrow 0^-} (|0 - h| - 0) / (-h) = -1$. Right-hand derivative = $\lim_{h \rightarrow 0^+} (h - 0) / h = 1$. LHD \neq RHD \rightarrow derivative does not exist at $x = 0$.

Q: What is marginal probability?

◆ In a joint distribution of two variables X and Y , the marginal probability $P(X = x)$ is obtained by summing $P(X = x, Y = y)$ over all values of y — 'marginalising out' Y .

■ **Pro Tip:** When asked a maths question, **THINK OUT LOUD**. Say: 'Let me approach this step by step...' Showing your reasoning impresses more than a rushed correct answer.